# Example Q4

Q4. The eight-point Discrete Fourier Transform (DFT) of the function:



is given by



1. Sketch the time sequence *x*[*n*].  
     
   *Answer*: point or stem plot with x-axis n and y-axis x[n].

[3 marks]

1. Use the DFT formula   
      
   to confirm that the value of *X*[4] = 2.   
     
   *Answer*: compute X[4] noting that the exponential term for m = 4 is -1.

[8 marks]

1. Use the DFT formula given in Q4(b) to confirm DC value of the signal.   
     
   *Answer*: compute X[0] but note that the exponential term is 1 for all n so this is simply the sum of x[n].

[3 marks]

1. Confirm that the correct value of *x*[1] is returned by the inverse DFT  
   .

[6 marks]

1. If we wanted to compute a 1024 point DFT from a sampled-data signal, how much speed-up could we expect to achieve using the FFT rather than the standard formula for the DFT?   
     
   *Answer*: FFT takes 9.8% of the time or is 102 times faster than DFT.

[3 marks]

1. Given that direct convolution of two digital signals takes order n2 floating point operations, is taking the FFT of the two 1024 point signals, multiplying them and taking the inverse FFT more efficient than direct convolution? If not, for how many samples, would using the FFT be more efficient than direct convolution?   
     
   *Answer*: convolution of two 1024 real signals takes 1,048,576 real operations. FFT of each signal takes 10,240 complex operations, multiplication of two signals in complex domain takes 1,024 complex multiplications, inverse FFT is 10,240 more. So FFT method is around 31,000 complex operations. Even if taking into account difference between complex and real arithmetic, FFT is order 10 times faster (and needs less memory) than convolution.

[2 marks]

Variations:

* Q4(a) Use Euler’s formula to determine the real and imaginary parts of *X*[*m*] as a function of *m* and *n*.
* Q4(a) sketch the power spectrum of *X*[*m*].
* Q4(b) use DFT to confirm a given value of *X*[*m*] for any value of *m*.
* Q4(d) compute *x*[*n*] from *X*[*m*] using inverse DFT from any value of *n.*